

Name

ANSWERS

Class



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Direct/inverse proportion

(9 – 1) Topic booklet

Higher

These questions have been collated from previous years GCSE Mathematics papers.

You must have: Ruler graduated in centimetres and millimetres,
protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided
 - *there may be more space than you need.*
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must show all your working out.
- If the question is a 1H question you are not allowed to use a calculator.
- If the question is a 2H or a 3H question, you may use a calculator to help you answer.

Information

- The marks for each question are shown in brackets
 - *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions
Write your answers in the space provided.
You must write down all the stages in your working.

6 A company orders a large number of plates from a factory.

It would take 30 hours to make all the plates using 4 machines.

How many machines are needed to make all the plates in 6 hours?



$$4 \text{ machines} = 30 \text{ hours}$$

$$1 \text{ machine} = 120 \text{ hours}$$

$$20 \text{ machines} = 6 \text{ hours}$$

20

November 2024 – Paper 3H

(Total for Question 6 is 2 marks)

6 At a depth of x metres, the temperature of the water in an ocean is $T^\circ\text{C}$.
At depths below 900 metres, T is inversely proportional to x .



T is given by

$$T = \frac{4500}{x}$$

(a) Work out the difference in the temperature of the water at a depth of 1200 metres and the temperature of the water at a depth of 2500 metres.

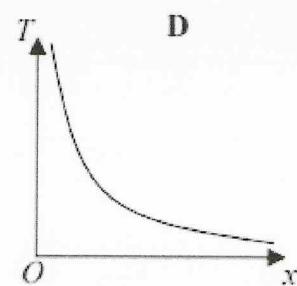
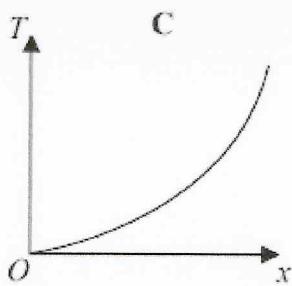
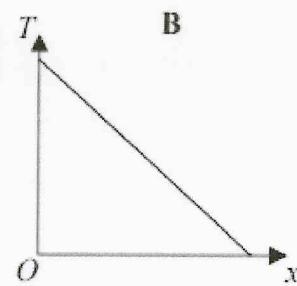
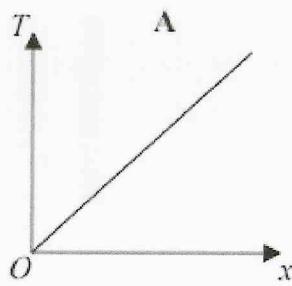
$$\frac{4500}{1200} = 3.75$$

$$\frac{4500}{2500} = 1.8$$

$$3.75 - 1.8 = 1.95$$

1.95 $^\circ\text{C}$
(3)

Here are four graphs.



One of the graphs could show that T is inversely proportional to x .

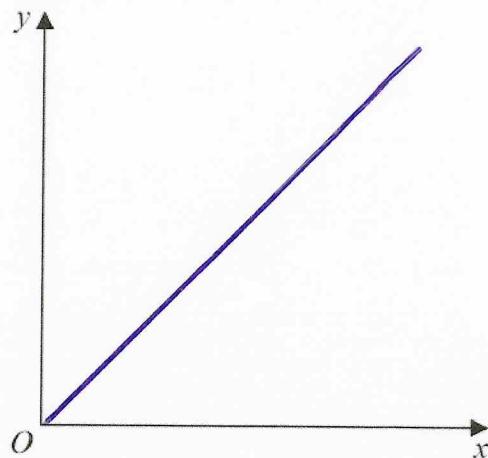
(b) Write down the letter of this graph.

D

(1)

8 (a) Using the axes below, sketch a graph to represent the statement

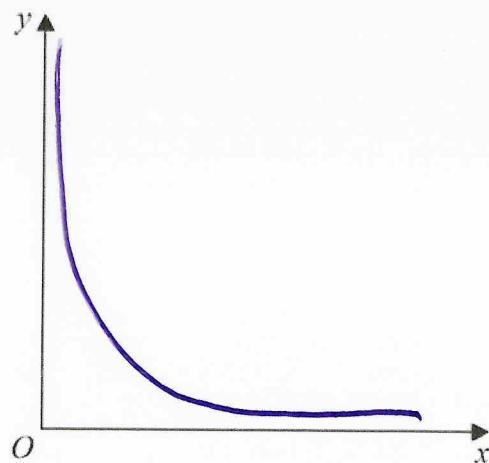
y is directly proportional to x



(1)

(b) Using the axes below, sketch a graph to represent the statement

y is inversely proportional to x



(1)

10 y is inversely proportional to x

When $x = 1.5$, $y = 36$

Find the value of y when $x = 6$

$$y \propto \frac{k}{x}$$

$$36 = \frac{k}{1.5}$$

$$54 = k$$

$$y = \frac{54}{x}$$

$$\frac{54}{6} = 9$$

9

Specimen 2 – Paper 1H

(Total for Question 10 is 3 marks)

12 f is inversely proportional to d^2

$f = 3.5$ when $d = 8$

$$f \propto \frac{k}{d^2}$$



(a) Find an equation for f in terms of d .

$$3.5 = \frac{k}{8^2}$$

$$3.5 \times 8^2 = k$$

$$224 = k$$

$$f = \frac{224}{d^2}$$

(b) Find the positive value of d when $f = 10$

Give your answer correct to 3 significant figures.

$$10 = \frac{224}{d^2}$$

$$d^2 = \frac{224}{10}$$

$$d = 4.73$$

(2)

June 2024 – Paper 2H

(Total for Question 12 is 4 marks)

$$d = \sqrt{22.4}$$

$$d = 4.732863826$$

13 p is inversely proportional to t

Complete the table of values.

t	100	25	20	2
p	1	4	5	50

$$p \propto \frac{k}{t}$$

$$p = \frac{100}{t}$$

$$1 = \frac{k}{100}$$

$$100 = k$$

13 The table shows a set of values for x and y .

x	1	2	3	4
y	9	$2\frac{1}{4}$	1	$\frac{9}{16}$

y is inversely proportional to the square of x .

(a) Find an equation for y in terms of x .

$$y \propto \frac{k}{x^2}$$

$$9 = \frac{k}{1^2}$$

$$9 = k$$

$$y = \frac{9}{x^2}$$

(2)

(b) Find the positive value of x when $y = 16$

$$16 = \frac{9}{x^2}$$

$$x^2 = \frac{9}{16}$$

$$\frac{3}{4}$$

(2)

May 2017 – Paper 1H

(Total for Question 13 is 4 marks)

$$x = \sqrt{\frac{9}{16}}$$

13 d is inversely proportional to c

When $c = 280$, $d = 25$



Find the value of d when $c = 350$

$$d \propto \frac{k}{c}$$

$$d = \frac{7000}{c}$$

$$25 = \frac{k}{280}$$

$$7000 = k$$

$$\frac{7000}{350} = 20$$

$$d = \underline{\quad 20 \quad}$$

Sample 1 – Paper 2H

(Total for Question 13 is 3 marks)

13 y is directly proportional to x .

$y = 24$ when $x = 1.5$

Work out the value of y when $x = 5$

$$y \propto kx$$

$$y = k \times x$$

$$24 = k \times 1.5$$

$$\frac{24}{1.5} = k$$

$$16 = k$$

$$y = 16 \times 5$$

$$16 \times 5 = 80$$

$$y = \underline{\quad 80 \quad}$$

June 2023 – Paper 1H

(Total for Question 13 is 3 marks)

14 y is proportional to x^2

$y = 3$ when $x = 0.5$



x is inversely proportional to w

$x = 2$ when $w = 0.2$

Find the value of y when $w = 2$

$$y \propto k \times x^2$$

$$y = k \times x^2$$

$$3 = k \times 0.5^2$$

$$\frac{3}{0.5^2} = k$$

$$12 = k$$

$$\boxed{y = 12x^2}$$

$$x \propto \frac{k}{w}$$

$$x = \frac{k}{w}$$

$$2 = \frac{k}{0.2}$$

$$0.4 = k$$

$$\boxed{x = \frac{0.4}{w}}$$

$$x = \frac{0.4}{2}$$

$$x = 0.2$$

$$y = 12 \times 0.2^2$$

$$y = 0.48$$

$$y = \dots \quad 0.48$$

November 2023 – Paper 2H

(Total for Question 14 is 5 marks)

14 y is inversely proportional to x^3

$y = 44$ when $x = a$



Show that $y = 5.5$ when $x = 2a$

$$y \propto \frac{k}{x^3}$$

$$y = \frac{44a^3}{x^3}$$

$$44 = \frac{k}{a^3}$$

$$x = 2a$$

$$44a^3 = k$$

$$\frac{44a^3}{(2a)^3} = \frac{44a^3}{8a^3} = \frac{44}{8} = 5.5$$

November 2018 – Paper 3H

(Total for Question 14 is 3 marks)

14 y is inversely proportional to d^2

When $d = 10$, $y = 4$

d is directly proportional to x^2

When $x = 2$, $d = 24$

Find a formula for y in terms of x .

Give your answer in its simplest form.

$$y \propto \frac{k}{d^2}$$

$$d \propto kx^2$$

$$24 = k \times 2^2$$

$$4 = \frac{k}{10^2}$$

$$\frac{24}{4} = k$$

$$400 = k$$

$$6 = k$$

$$\boxed{y = \frac{400}{d^2}}$$

$$\boxed{d = 6x^2}$$

$$y = \frac{400}{(6x^2)^2} = \frac{400}{36x^4} = \frac{200}{18x^4} = \frac{100}{9x^4}$$

$$y = \frac{100}{9x^4}$$

14 D is directly proportional to the cube of n .

Mary says that when n is doubled, the value of D is multiplied by 6

Mary is wrong.
Explain why.



$$D = k \times n^3$$

When n is doubled, the value of n^3 becomes $(2n)^3$
which equals $8n^3$.

This means D is multiplied by 8 not 6.

(1)

Specimen 2 – Paper 2H

(Total for Question 14 is 1 mark)

15 A pendulum of length L cm has time period T seconds.

T is directly proportional to the square root of L .



The length of the pendulum is increased by 40%. $\rightarrow 100\% + 40\% = 140\%$

Work out the percentage increase in the time period.

$= 1.4$

$$T \propto k \times \sqrt{L}$$

$$T = k \sqrt{L}$$

$$T = k \sqrt{1.4L}$$

$1.18 = 18\% \text{ increase}$

$$T = k \sqrt{1.4} \sqrt{L}$$

$$T = k \times 1.183215 \times \sqrt{L}$$

18
%

Specimen 1 – Paper 2H

(Total for Question 15 is 3 marks)

16 y is directly proportional to $\sqrt[3]{x}$

$$y = 1 \frac{1}{6} \text{ when } x = 8$$

Find the value of y when $x = 64$

$$y \propto k \times \sqrt[3]{x}$$

$$1 \frac{1}{6} = k \times \sqrt[3]{8}$$

$$1 \frac{1}{6} = k \times 2$$

$$\frac{7}{12} = k$$

$$y = \frac{7}{12} \times \sqrt[3]{x}$$

$$y = \frac{7}{12} \times \sqrt[3]{64}$$

$$= \frac{7}{12} \times 4$$

$$= \frac{28}{12}$$

$$= 2 \frac{4}{12}$$

$$2 \frac{1}{3}$$

November 2017 – Paper 1H

(Total for Question 16 is 3 marks)

17 y is directly proportional to the square root of t .

$$y = 15 \text{ when } t = 9$$

t is inversely proportional to the cube of x .

$$t = 8 \text{ when } x = 2$$

Find a formula for y in terms of x .

Give your answer in its simplest form.

$$y = k \times \sqrt{t}$$

$$t = \frac{k}{x^3}$$

$$15 = k \times \sqrt{9}$$

$$8 = \frac{k}{2^3}$$

$$15 = k \times 3$$

$$8 = \frac{k}{8}$$

$$5 = k$$

$$64 = k$$

$$y = 5 \times \sqrt{t}$$

$$t = \frac{64}{x^3}$$

$$y = 5 \times \sqrt{\frac{64}{x^3}}$$

$$y = 5 \times \frac{8}{\sqrt{x^3}}$$

$$y = \frac{40}{\sqrt{x^3}}$$

$$y = \frac{40}{\sqrt{x^3}}$$

17 x is directly proportional to the square of y .
 y is directly proportional to the cube of z .



$$z = 2 \text{ when } x = 32$$

Find a formula for x in terms of z .

$$x = ky^2$$

$k = \text{constant}$

$$y = cz^3$$

$c = \text{constant}$

$$x = k(cz^3)^2$$

$$x = k(c^2 z^6)$$

$$x = kc^2 z^6$$

$$32 = kc^2 \times 2^6$$

$$\frac{32}{2^6} = kc^2$$

$$\frac{1}{2} = kc^2$$

$$x = \frac{1}{2} z^6$$

18 x is proportional to \sqrt{y} where $y > 0$

y is increased by 44% $\rightarrow 100\% + 44\% = 144\%$

Work out the percentage increase in x .

$$= 1.44$$

$$x \propto \sqrt{y}$$

$$x = k\sqrt{1.44y}$$

$$x = k\sqrt{1.44}\sqrt{y}$$

$$x = k \times 1.2 \times \sqrt{y}$$

$$1.2 = 120\%$$

20

0

November 2020 – Paper 1H

(Total for Question 18 is 3 marks)

20 h is inversely proportional to p

p is directly proportional to \sqrt{t}

Given that $h = 10$ and $t = 144$ when $p = 6$
find a formula for h in terms of t

$$h \propto \frac{k}{p}$$

$$p \propto k \times \sqrt{t}$$

$$10 = \frac{k}{6}$$

$$6 = k \sqrt{144}$$

$$60 = k$$

$$6 = k \times 12$$

$$\frac{1}{2} = k$$

$$h = \frac{60}{p}$$

$$p = \frac{1}{2} \sqrt{t}$$

$$h = \frac{60}{\frac{1}{2} \sqrt{t}}$$

$$h = \frac{120}{\sqrt{t}}$$

$$h = \frac{120}{\sqrt{t}}$$